Que. No.	Sub. Que.	Model Answers	Marks	Total Marks

Important Instructions to the Examiners:

1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		**Attempt any TEN of the following:**		
	a)	If $\frac{10}{3+4i}=a+ib$, find a and b.		
	Ans.	$\frac{10}{3+4i}=a+ib$		
		$\therefore \frac{10}{3+4i}\times\frac{3-4i}{3-4i}=a+ib$	½	
		$\therefore \frac{30-40i}{3^2-(4i)^2}=a+ib$		
		$\therefore \frac{30-40i}{9+16}=a+ib$		
		$\therefore \frac{30-40i}{25}=a+ib$	½	
		$\therefore \frac{30}{25}-\frac{40}{25}i=a+ib$		
		$\therefore \frac{6}{5}-\frac{8}{5}i=a+ib$		
		$\therefore a=\frac{6}{5} \quad and \quad b=-\frac{8}{5}$	½ +½	**2**
		---		
	b)	If $z=3+4i$, find $z^2-6z+25$		
	Ans.	$z^2-6z+25=(3+4i)^2-6(3+4i)+25$	½	
		$=3^2+2\cdot3\cdot4i+4i^2-18-24i+25$	1	
		$=9+24i-16-18-24i+25$	½	**2**
		$=0$		
		OR		
		$z^2=(3+4i)^2=9+24i-16=-7+24i$	1	
		$-6z=-6(3+4i)=-18-24i$	½	
		$\therefore z^2-6z+25=-7+24i-18-24i+25$		
		$=0$	½	**2**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	c)	If $f(x)=x^2+6x+10$, find $f(2)+f(-2)$		
	Ans.	$f(x)=x^2+6x+10$		
		$\therefore f(2)=2^2+6(2)+10=26$	½	
		$f(-2)=(-2)^2+6(-2)+10=2$	½	
		$\therefore f(2)+f(-2)=28$	1	**2**
		OR		
		$f(2)+f(-2)=\left[2^2+6(2)+10\right]+\left[(-2)^2+6(-2)+10\right]$	1	
		$=28$	1	**2**
		--		
	d)	If $f(x)=\dfrac{a^x+a^{-x}}{2}$, prove that the function is even function.		
	Ans.	$f(-x)=\dfrac{a^{-x}+a^{-(-x)}}{2}$	½	
		$=\dfrac{a^{-x}+a^x}{2}$	½	
		$=f(x)$	½	
		$\therefore f(x)$ is an even function.	½	**2**
		--		
	e)	Evaluate $\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}$		
	Ans.	$\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}=\lim\limits_{x\to 3}\dfrac{(x-3)(x+3)}{x-3}$	½	
		$=\lim\limits_{x\to 3}(x+3)$	½	
		$=3+3$	½	
		$=6$	½	**2**
		OR		
		$\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}=\lim\limits_{x\to 3}\dfrac{x^2-3^2}{x-3}$	½	
		$=2\times 3^{2-1}$	1	
		$=6$	½	**2**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	f)	Evaluate $\lim_{x\to 0}\frac{1-\cos x}{x^2}$		
	Ans.	$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{2\sin^2\left(\frac{x}{2}\right)}{x^2}$	½	
		$=\lim_{x\to 0}2\left(\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\times\frac{1}{2}\right)^2$	½	
		$=2\left(1\times\frac{1}{2}\right)^2$	½	
		$=\frac{1}{2}\quad or\quad 0.5$	½	**2**
		OR		
		$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{1-\cos x}{x^2}\times\frac{1+\cos x}{1+\cos x}$	½	
		$=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2}\times\frac{1}{1+\cos x}$		
		$=\lim_{x\to 0}\frac{\sin^2 x}{x^2}\times\frac{1}{1+\cos x}$	½	
		$=\lim_{x\to 0}\left[\frac{\sin x}{x}\right]^2\times\frac{1}{1+\cos x}$		
		$=[1]^2\times\frac{1}{1+\cos 0}$	½	
		$=\frac{1}{2}$	½	**2**
		--		
	g)	Evaluate $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x$		
	Ans.	$Put\ \ x+1=t\qquad \therefore t\to\infty$		
		$\therefore\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x=\lim_{t\to\infty}\left(\frac{t-1}{t}\right)^{t-1}$	½	
		$=\lim_{t\to\infty}\left(1-\frac{1}{t}\right)^{t-1}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		$=\lim_{t\to\infty}\left(1-\frac{1}{t}\right)^{t}\times\left(1-\frac{1}{t}\right)^{-1}$	½	
		$=\lim_{t\to\infty}\left(1-\frac{1}{t}\right)^{-t\times-1}\times\left(1-\frac{1}{t}\right)^{-1}$		
		$=e^{-1}\times(1-0)^{-1}$	½	
		$=e^{-1}$	½	**2**
		--		
	h)	If $y=e^{x}\cdot\sin x$, find $\frac{dy}{dx}$		
	Ans.	$\therefore\frac{dy}{dx}=e^{x}\cdot\frac{d}{dx}(\sin x)+\sin x\cdot\frac{d}{dx}\left(e^{x}\right)$	½	
		$=e^{x}\cdot\cos x+\sin x\cdot e^{x}$	1	
		$=e^{x}\left(\cos x+\sin x\right)$	½	**2**
		--		
	i)	If $y=\tan^{-1}\left(\frac{a+x}{1-ax}\right)$, find $\frac{dy}{dx}$		
	Ans.	$y=\tan^{-1}\left(\frac{a+x}{1-ax}\right)$		
		$Put\ a=\tan A,\quad x=\tan B$		
		$\therefore y=\tan^{-1}\left(\frac{\tan A+\tan B}{1-\tan A\cdot\tan B}\right)$		
		$=\tan^{-1}\left[\tan\left(A+B\right)\right]$		
		$=A+B$		
		$=\tan^{-1}a+\tan^{-1}x$	1	
		$\therefore\frac{dy}{dx}=\frac{1}{1+x^{2}}$	1	**2**
		OR		
		[The same can be solved by applying directly the result $\tan^{-1}a+\tan^{-1}b=\tan^{-1}\left(\frac{a+b}{1-ab}\right)$. This is also allowed.]		
		$y=\tan^{-1}\left(\frac{a+x}{1-ax}\right)$		
		$=\tan^{-1}a+\tan^{-1}x$	1	
		$\therefore\frac{dy}{dx}=\frac{1}{1+x^{2}}$	1	**2**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	j)	If $x=a\sec t$ and $y=b\tan t$, find $\frac{dy}{dx}$		
	Ans.	$x=a\sec t$		
		$\therefore \frac{dx}{dt}=a\sec t\tan t$	½	
		$y=b\tan t$		
		$\therefore \frac{dy}{dt}=b\sec^2 t$	½	
		$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}$		
		$=\frac{b\sec^2 t}{a\sec t\tan t}$	½	
		$=\frac{b}{a\sin t}$	½	**2**
	k)	Prove that the root of the equation $x^3-x-4=0$ lies between 0 and 2.		
	Ans.	$x^3-x-4=0$		
		$\therefore f(x)=x^3-x-4$		
		$\therefore f(0)=-4$	1	
		$f(2)=2$	½	
		$\therefore$ root lies between 0 and 2.	½	**2**
	l)	Find the first iteration by using Jacobi's method for the following equations: $4x+y+3z=17,\ x+5y+z=14,\ 2x-y+8z=12$		
	Ans.	$\therefore x=\frac{1}{4}(17-y-3z)$		
		$y=\frac{1}{5}(14-x-z)$	1	
		$z=\frac{1}{8}(12-2x+y)$		
		Starting with $x_0=0=y_0=z_0$		
		$x_1=4.25$	1	
		$y_1=2.8$		**2**
		$z_1=1.5$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		**Attempt any FOUR of the following:**		
	a)	If $f(x)=\tan x$ then show that $f(2x)=\frac{2f(x)}{1-f^2(x)}$		
	Ans.	$f(2x)=\tan(2x)$	1	
		$=\frac{2\tan x}{1-\tan^2 x}$	1½	
		$=\frac{2f(x)}{1-f^2(x)}$	1½	**4**
		---		
	b)	Simplify using DeMovire's theorem $\frac{(\cos 3\theta+i\sin 3\theta)^4(\cos 5\theta-i\sin 5\theta)^{\frac{4}{5}}}{\left(\cos\frac{3}{5}\theta+i\sin\frac{3}{5}\theta\right)^5\left(\cos\frac{4}{5}\theta+i\sin\frac{4}{5}\theta\right)^{10}}$		
	Ans.	$\frac{(\cos 3\theta+i\sin 3\theta)^4(\cos 5\theta-i\sin 5\theta)^{\frac{4}{5}}}{\left(\cos\frac{3}{5}\theta+i\sin\frac{3}{5}\theta\right)^5\left(\cos\frac{4}{5}\theta+i\sin\frac{4}{5}\theta\right)^{10}}$		
		$=\frac{(\cos\theta+i\sin\theta)^{3\times4}(\cos\theta+i\sin\theta)^{-5\times\frac{4}{5}}}{(\cos\theta+i\sin\theta)^{\frac{3}{5}\times5}(\cos\theta+i\sin\theta)^{\frac{4}{5}\times10}}$	½+½+ ½+½	
		$=\frac{(\cos\theta+i\sin\theta)^{12}(\cos\theta+i\sin\theta)^{-4}}{(\cos\theta+i\sin\theta)^{3}(\cos\theta+i\sin\theta)^{8}}$	1	
		$=(\cos\theta+i\sin\theta)^{12-4-3-8}$		
		$=(\cos\theta+i\sin\theta)^{-3}$	½	
		$=\cos 3\theta-i\sin 3\theta$	½	**4**
		OR		
		$(\cos 3\theta+i\sin 3\theta)^4=(\cos\theta+i\sin\theta)^{3\times4}=(\cos\theta+i\sin\theta)^{12}$	½	
		$(\cos 5\theta-i\sin 5\theta)^{\frac{4}{5}}=(\cos\theta+i\sin\theta)^{-5\times\frac{4}{5}}=(\cos\theta+i\sin\theta)^{-4}$	½	
		$\left(\cos\frac{3}{5}\theta+i\sin\frac{3}{5}\theta\right)^5=(\cos\theta+i\sin\theta)^{\frac{3}{5}\times5}=(\cos\theta+i\sin\theta)^{3}$	½	
		$\left(\cos\frac{4}{5}\theta+i\sin\frac{4}{5}\theta\right)^{10}=(\cos\theta+i\sin\theta)^{\frac{4}{5}\times10}=(\cos\theta+i\sin\theta)^{8}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$\dfrac{(\cos 3\theta+i\sin 3\theta)^{4}(\cos 5\theta-i\sin 5\theta)^{\frac{4}{5}}}{\left(\cos\frac{3}{5}\theta+i\sin\frac{3}{5}\theta\right)^{5}\left(\cos\frac{4}{5}\theta+i\sin\frac{4}{5}\theta\right)^{10}}$		
		$=\dfrac{(\cos\theta+i\sin\theta)^{12}(\cos\theta+i\sin\theta)^{-4}}{(\cos\theta+i\sin\theta)^{3}(\cos\theta+i\sin\theta)^{8}}$	1	
		$=(\cos\theta+i\sin\theta)^{12-4-3-8}$		
		$=(\cos\theta+i\sin\theta)^{-3}$	½	
		$=\cos 3\theta-i\sin 3\theta$	½	**4**
	c)	Separate into real and imaginary part of $\sin(x+iy)$		
	Ans.	$\sin(x+iy)=\sin x\cos iy+\cos x\sin iy$	2	
		$=\sin x\cosh y+i\cos x\sinh y$	2	**4**
	d)	Express in polar form $1-\sqrt{3}i$		
	Ans.	Let $z=1-\sqrt{3}i$		
		$\therefore r=\sqrt{(1)^{2}+\left(-\sqrt{3}\right)^{2}}=2$	1	
		$\theta=\tan^{-1}\left(\dfrac{-\sqrt{3}}{1}\right)=-60^{\circ}\ \ or\ \ -\dfrac{\pi}{3}$	1	
		$\therefore z=r(\cos\theta+i\sin\theta)$		
		$=2\left[\cos(-60^{\circ})+i\sin(-60^{\circ})\right]\quad or\quad 2\left[\cos\left(-\dfrac{\pi}{3}\right)+i\sin\left(-\dfrac{\pi}{3}\right)\right]$	1½	
		$=2\left[\cos 60^{\circ}-i\sin 60^{\circ}\right]\quad or\quad 2\left[\cos\dfrac{\pi}{3}-i\sin\dfrac{\pi}{3}\right]$	½	**4**
		OR		
		$\therefore r=\sqrt{(1)^{2}+\left(-\sqrt{3}\right)^{2}}=2$	1	
		$\theta=360^{\circ}-\tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right)\quad or\quad 2\pi-\tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right)$		
		$=300^{\circ}\quad or\quad \dfrac{5\pi}{3}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)				
		$\therefore z = r\left(\cos\theta + i\sin\theta\right)$		
		$= 2\left[\cos 300^o + i\sin 300^o\right]$ *or* $2\left[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right]$	2	**4**
		--		
	e)	Show that $(1+i)^{12} + (1-i)^{12} = -128$		
	Ans.	$(1+i)^{12} = \left[(1+i)^2\right]^6$	½	
		$= \left[1 + 2i + i^2\right]^6$		
		$= [1 + 2i - 1]^6$	½	
		$= [2i]^6$	½	
		$= 2^6 i^6$		
		$= -64$	½	
		$\therefore (1-i)^{12} = -64$	1	
		$\therefore (1+i)^{12} + (1-i)^{12} = -128$	1	**4**
		OR		
		$\therefore (1+i)^{12} + (1-i)^{12} = \left[(1+i)^2\right]^6 + \left[(1-i)^2\right]^6$	½+½	
		$= \left[1 + 2i + i^2\right]^6 + \left[1 - 2i + i^2\right]^6$		
		$= [1 + 2i - 1]^6 + [1 - 2i - 1]^6$	½+½	
		$= [2i]^6 + [-2i]^6$	½+½	
		$= -64 - 64$		
		$= -128$	1	**4**
		OR		
		$\therefore r = \sqrt{1^2 + 1^2} = \sqrt{2}$	½	
		$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$	½	
		$\therefore 1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$	½	
		$\therefore (1+i)^{12} = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{12}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$=\sqrt{2}^{12}\left(\cos 12\times\frac{\pi}{4}+i\sin 12\times\frac{\pi}{4}\right)$		
		$=64\left(\cos 3\pi+i\sin 3\pi\right)$	½	
		$\therefore \left(1-i\right)^{12}=64\left(\cos 3\pi-i\sin 3\pi\right)$	½	
		$\therefore \left(1+i\right)^{12}+\left(1-i\right)^{12}=64\left(\cos 3\pi+i\sin 3\pi\right)+64\left(\cos 3\pi-i\sin 3\pi\right)$		
		$=128\cos 3\pi$	1	
		$=-128$	½	**4**
		--		
	f)	If $f\left(x\right)=\log\left(\frac{x+1}{x-1}\right)$ then show that $f\left(\frac{1+x^2}{2x}\right)=2f\left(x\right)$		
	Ans.	$\therefore f\left(\frac{1+x^2}{2x}\right)=\log\left(\frac{\frac{1+x^2}{2x}+1}{\frac{1+x^2}{2x}-1}\right)$	1	
		$=\log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$	1	
		$=\log\left[\frac{\left(x+1\right)^2}{\left(x-1\right)^2}\right]$	½	
		$=\log\left(\frac{x+1}{x-1}\right)^2$	½	
		$=2\log\left(\frac{x+1}{x-1}\right)$	½	
		$=2f(x)$	½	**4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		**Attempt any FOUR of the following:**		
	a)	Find $f(t)$, if $f(x)=\frac{2x+5}{3x-4}$ and $t=\frac{5+4x}{3x-2}$		
	Ans.	$f(t)=\frac{2t+5}{3t-4}$	½	
		$=\frac{2\left(\frac{5+4x}{3x-2}\right)+5}{3\left(\frac{5+4x}{3x-2}\right)-4}$	½	
		$=\frac{\frac{2(5+4x)+5(3x-2)}{3x-2}}{\frac{3(5+4x)-4(3x-2)}{3x-2}}$ *or* $\frac{2(5+4x)+5(3x-2)}{3(5+4x)-4(3x-2)}$	1	
		$=\frac{10+8x+15x-10}{15+12x-12x+8}$	1	
		$=\frac{23x}{23}$	½	
		$=x$	½	**4**
		--		
	b)	Evaluate $\lim_{x\to 0}\frac{3^x+3^{-x}-2}{x^2}$		
	Ans.	$\lim_{x\to 0}\frac{3^x+3^{-x}-2}{x^2}=\lim_{x\to 0}\frac{3^x+\frac{1}{3^x}-2}{x^2}$	½	
		$=\lim_{x\to 0}\frac{\frac{\left(3^x\right)^2+1-2\left(3^x\right)}{3^x}}{x^2}$	½	
		$=\lim_{x\to 0}\frac{\left(3^x-1\right)^2}{x^2}\times\frac{1}{3^x}$	1	
		$=\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)^2\times\frac{1}{3^x}$	½	
		$=(\log 3)^2\times\frac{1}{3^0}$	1	
		$=(\log 3)^2$	½	**4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)				
	c)	If $f\left(x\right)=x^2-3x+4$ and $f\left(1-x\right)=f\left(2x+1\right)$		
	Ans.	$f\left(1-x\right)=f\left(2x+1\right)$		
		$\therefore\left(1-x\right)^2-3\left(1-x\right)+4=\left(2x+1\right)^2-3\left(2x+1\right)+4$	½ + ½	
		$\therefore 1-2x+x^2-3+3x+4=4x^2+4x+1-6x-3+4$	½ + ½	
		$\therefore x^2+x+2=4x^2-2x+2$		
		$\therefore -3x^2+3x=0 \qquad or \qquad 3x^2-3x=0$	1	
		$\therefore x=0,\ 1$	½ + ½	**4**
		OR		
		$f\left(1-x\right)=\left(1-x\right)^2-3\left(1-x\right)+4$	½	
		$=1-2x+x^2-3+3x+4$		
		$=x^2+x+2$	½	
		$f\left(2x+1\right)=\left(2x+1\right)^2-3\left(2x+1\right)+4$	½	
		$=4x^2+4x+1-6x-3+4$		
		$=4x^2-2x+2$	½	
		$But \quad f\left(1-x\right)=f\left(2x+1\right)$		
		$\therefore x^2+x+2=4x^2-2x+2$		
		$\therefore -3x^2+3x=0 \qquad or \qquad 3x^2-3x=0$	1	
		$\therefore x=0,\ 1$	½ + ½	**4**
		---		
	d)	Evaluate $\lim\limits_{x\to 3}\dfrac{x^3-7x^2+15x-9}{x^3-4x^2-3x+18}$		
	Ans.	$\lim\limits_{x\to 3}\dfrac{x^3-7x^2+15x-9}{x^3-4x^2-3x+18}=\lim\limits_{x\to 3}\dfrac{(x-3)\left(x-3\right)\left(x-1\right)}{(x-3)\left(x-3\right)\left(x+2\right)}$	1½	
		$=\lim\limits_{x\to 3}\dfrac{x-1}{x+2}$	1	
		$=\dfrac{3-1}{3+2}$	1	
		$=\dfrac{2}{5} \quad or \quad 0.4$	½	**4**
		OR		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)				
		$\lim\limits_{x\to 3}\frac{x^3-7x^2+15x-9}{x^3-4x^2-3x+18}=\lim\limits_{x\to 3}\frac{(x-3)\left(x^2-4x+3\right)}{(x-3)\left(x^2-x-6\right)}$	½	
		$=\lim\limits_{x\to 3}\frac{x^2-4x+3}{x^2-x-6}$	½	
		$=\lim\limits_{x\to 3}\frac{(x-3)(x-1)}{(x-3)(x+2)}$	½	
		$=\lim\limits_{x\to 3}\frac{x-1}{x+2}$	1	
		$=\frac{3-1}{3+2}$	1	
		$=\frac{2}{5}\quad or\quad 0.4$	½	**4**
		---		
	e)	Evaluate $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$		
	Ans.	$\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$		
		$=\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)\times\frac{\sqrt{x^2+x+1}+x}{\sqrt{x^2+x+1}+x}$	½	
		$=\lim\limits_{x\to\infty}\frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x}$	½	
		$=\lim\limits_{x\to\infty}\frac{x+1}{\sqrt{x^2+x+1}+x}$	½	
		$=\lim\limits_{x\to\infty}\frac{\frac{x+1}{x}}{\frac{\sqrt{x^2+x+1}+x}{x}}$		
		$=\lim\limits_{x\to\infty}\frac{1+\frac{1}{x}}{\sqrt{\frac{x^2+x+1}{x^2}}+\frac{x}{x}}$	½	
		$=\lim\limits_{x\to\infty}\frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}$	½	
		$=\frac{1+0}{\sqrt{1+0+0}+1}$	1	
		$=\frac{1}{2}$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	f)	Evaluate $\lim\limits_{x\to0}\dfrac{2\sin x-\sin 2x}{x^3}$		
	Ans.	$\lim\limits_{x\to0}\dfrac{2\sin x-\sin 2x}{x^3}=\lim\limits_{x\to0}\dfrac{2\sin x-2\sin x\cos x}{x^3}$	½	
		$=\lim\limits_{x\to0}\dfrac{2\sin x\left(1-\cos x\right)}{x^3}$		
		$=\lim\limits_{x\to0}\dfrac{2\sin x\left(2\sin^2\left(\dfrac{x}{2}\right)\right)}{x^3}$	1	
		$=\lim\limits_{x\to0}\dfrac{4\sin x\cdot\sin^2\left(\dfrac{x}{2}\right)}{x^3}$	½	
		$=\lim\limits_{x\to0}4\cdot\dfrac{\sin x}{x}\cdot\left(\dfrac{\sin\left(\dfrac{x}{2}\right)}{\dfrac{x}{2}}\times\dfrac{1}{2}\right)^2$	1	
		$=4\cdot1\cdot\left(1\times\dfrac{1}{2}\right)^2$	½	
		$=1$	½	**4**
		OR		
		$\lim\limits_{x\to0}\dfrac{2\sin x-\sin 2x}{x^3}=\lim\limits_{x\to0}\dfrac{2\sin x-2\sin x\cos x}{x^3}$	½	
		$=\lim\limits_{x\to0}\dfrac{2\sin x\left(1-\cos x\right)}{x^3}$		
		$=\lim\limits_{x\to0}\dfrac{2\sin x\left(1-\cos x\right)}{x^3}\times\dfrac{1+\cos x}{1+\cos x}$	½	
		$=\lim\limits_{x\to0}\dfrac{2\sin x\left(1-\cos^2 x\right)}{x^3}\times\dfrac{1}{1+\cos x}$		
		$=\lim\limits_{x\to0}\dfrac{2\sin x\left(\sin^2 x\right)}{x^3}\times\dfrac{1}{1+\cos x}$	½	
		$=\lim\limits_{x\to0}\dfrac{2\sin^3 x}{x^3}\times\dfrac{1}{1+\cos x}$	½	
		$=\lim\limits_{x\to0}2\left(\dfrac{\sin x}{x}\right)^3\times\dfrac{1}{1+\cos x}$	1	
		$=2\left(1\right)^3\times\dfrac{1}{1+\cos 0}$	½	
		$=1$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Attempt any FOUR of the following:**		
	a)	Find $\frac{dy}{dx}$, if $y=\cos^{-1}\left(2x^2-1\right)$		
	Ans.	$Put\ \ x=\cos\theta$	½	
		$\therefore y=\cos^{-1}\left(2x^2-1\right)$		
		$=\cos^{-1}\left(2\cos^2\theta-1\right)$	½	
		$=\cos^{-1}\left(\cos 2\theta\right)$	½	
		$=2\theta$	½	
		$=2\cos^{-1}x$	1	
		$\therefore \frac{dy}{dx}=-2\cdot\frac{1}{\sqrt{1-x^2}}$	1	**4**
		OR		
		$Put\ \ x=\sin\theta$	½	
		$\therefore y=\cos^{-1}\left(2x^2-1\right)$		
		$=\cos^{-1}\left(2\sin^2\theta-1\right)$	½	
		$=\cos^{-1}\left(-\cos 2\theta\right)$	½	
		$=\pi-2\theta$	½	
		$=\pi-2\sin^{-1}x$	1	
		$\therefore \frac{dy}{dx}=-2\cdot\frac{1}{\sqrt{1-x^2}}$	1	**4**
		OR		
		$y=\cos^{-1}\left(2x^2-1\right)$		
		$\therefore \cos y=2x^2-1$	1	
		$\therefore -\sin y\frac{dy}{dx}=4x$	2	
		$\therefore \frac{dy}{dx}=-\frac{4x}{\sin y}$	1	**4**
		OR		
		$y=\cos^{-1}\left(2x^2-1\right)$		
		$\therefore \frac{dy}{dx}=-\frac{1}{\sqrt{1-\left(2x^2-1\right)^2}}\cdot\frac{d}{dx}\left(2x^2-1\right)$	1	
		$=-\frac{1}{\sqrt{1-\left(4x^4-4x^2+1\right)}}\cdot\left(4x\right)$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$=-\frac{1}{\sqrt{-4x^4+4x^2}}\cdot(4x)$		
		$=-\frac{1}{\sqrt{4x^2(1-x^2)}}\cdot(4x)$		
		$=-\frac{1}{2x\sqrt{1-x^2}}\cdot(4x)$	1	
		$=-\frac{2}{\sqrt{1-x^2}}$	1	**4**
		---		
	b)	If $x^2+y^2-xy=0$, find $\frac{dy}{dx}$.		
	Ans.	$x^2+y^2-xy=0$		
		$\therefore 2x+2y\frac{dy}{dx}-\left(x\frac{dy}{dx}+y\right)=0$	1	
		$\therefore 2x+2y\frac{dy}{dx}-x\frac{dy}{dx}-y=0$	1	
		$\therefore 2x-y+(2y-x)\frac{dy}{dx}=0$ *or* $(2y-x)\frac{dy}{dx}=-2x+y$	1	
		$\therefore \frac{dy}{dx}=-\frac{2x-y}{2y-x}$ *or* $\frac{dy}{dx}=\frac{-2x+y}{2y-x}$	1	**4**
		---		
	c)	If $x=a(1+\cos\theta),\quad y=a(1-\cos\theta)$, find $\frac{dy}{dx}$.		
	Ans.	$x=a(1+\cos\theta)$		
		$\therefore \frac{dx}{d\theta}=a(-\sin\theta)=-a\sin\theta$	1	
		$y=a(1-\cos\theta)$		
		$\therefore \frac{dy}{d\theta}=a\sin\theta$	1	
		$\therefore \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$		
		$=\frac{a\sin\theta}{-a\sin\theta}$	1	
		$=-1$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**OR**		
		$x=a(1+\cos\theta)=a+a\cos\theta$		
		$\therefore a\cos\theta=x-a$	½	
		$y=a(1-\cos\theta)=a-a\cos\theta$		
		$\therefore a\cos\theta=a-y$	½	
		$\therefore x-a=a-y$	1	
		$\therefore 1=-\frac{dy}{dx}$	1	
		$\therefore \frac{dy}{dx}=-1$	1	**4**
		--		
	d)	Using first principle, find derivative of $f(x)=\tan x$.		
	Ans.	$f(x)=\tan x$		
		$\therefore f(x+h)=\tan(x+h)$		
		$\therefore \frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$		
		$=\lim_{h\to 0}\frac{\tan(x+h)-\tan x}{h}$	1	
		$=\lim_{h\to 0}\frac{\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}}{h}$	½	
		$=\lim_{h\to 0}\left[\frac{\frac{\sin(x+h)\cos x-\sin x\cos(x+h)}{\cos(x+h)\cos x}}{h}\right]$	½	
		$=\lim_{h\to 0}\left[\frac{\sin(x+h-x)}{\cos(x+h)\cos x}\times\frac{1}{h}\right]$	½	
		$=\lim_{h\to 0}\left[\frac{\sin h}{\cos(x+h)\cos x}\times\frac{1}{h}\right]$		
		$=\lim_{h\to 0}\left[\frac{1}{\cos(x+h)\cos x}\times\frac{\sin h}{h}\right]$	½	
		$=\frac{1}{\cos x\cos x}\times 1$	½	
		$=\sec^2 x$	½	**4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	e)	If u and v are differentiable functions of x and $y=u+v$, then prove that $\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$.		
	Ans.	Let δx be infinitesimal increment in x and δy, δu, δv be corresponding infinitesimal increments in y, u, v.	½	
		$\therefore y+\delta y=(u+\delta u)+(v+\delta v)$	½	
		$\therefore \delta y=(u+\delta u)+(v+\delta v)-y$		
		$=u+\delta u+v+\delta v-(u+v)$	½	
		$=\delta u+\delta v$		
		$\therefore \frac{\delta y}{\delta x}=\frac{\delta u+\delta v}{\delta x}=\frac{\delta u}{\delta x}+\frac{\delta v}{\delta x}$	½	
		$\therefore \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\left[\frac{\delta u}{\delta x}+\frac{\delta v}{\delta x}\right]$	1	
		$\therefore \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\frac{\delta u}{\delta x}+\lim_{\delta x\to 0}\frac{\delta v}{\delta x}$		
		$\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	1	**4**
		--		
	f)	If $x^{y}=e^{x-y}$, prove $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^{2}}$		
	Ans.	Given $x^{y}=e^{x-y}$		
		$\therefore y\log x=x-y$	½	
		$\therefore y\log x+y=x$		
		$\therefore y(\log x+1)=x$		
		$\therefore y=\frac{x}{\log x+1}$	½	
		$\therefore \frac{dy}{dx}=\frac{(\log x+1)\frac{d}{dx}(x)-x\frac{d}{dx}(\log x+1)}{(\log x+1)^{2}}$		
		$\therefore \frac{dy}{dx}=\frac{(\log x+1)\cdot 1-x\left(\frac{1}{x}+0\right)}{(\log x+1)^{2}}$	1	
		$\therefore \frac{dy}{dx}=\frac{\log x+1-1}{(\log x+1)^{2}}$	1	
		$\therefore \frac{dy}{dx}=\frac{\log x}{(\log x+1)^{2}}$	1	**4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Attempt any FOUR of the following:**		
	a)	Evaluate $\lim_{x\to3}\frac{\log x-\log 3}{x-3}$		
	Ans.	$\lim_{x\to3}\left[\frac{\log x-\log 3}{x-3}\right]$ $\left\vert\begin{array}{l} Let\ x=3+h \quad or \quad x-3=h \\ as\ x\to3,\ h\to0 \end{array}\right.$	1	
		$=\lim_{h\to0}\left[\frac{\log(3+h)-\log 3}{3+h-3}\right]$		
		$=\lim_{h\to0}\frac{1}{h}\log\left(\frac{3+h}{3}\right)$	1	
		$=\lim_{h\to0}\log\left(1+\frac{h}{3}\right)^{1/h}$	½	
		$=\lim_{h\to0}\log\left(1+\frac{h}{3}\right)^{3/h\times1/3}$	½	
		$=\log e^{\frac{1}{3}}$	½	
		$=\frac{1}{3}\log e$		
		$=\frac{1}{3}$	½	**4**
		---		
	b)	Evaluate $\lim_{x\to0}\frac{(5^x-1)\tan x}{\sqrt{x^2+16}-4}$		
	Ans.	$\lim_{x\to0}\frac{(5^x-1)\tan x}{\sqrt{x^2+16}-4}=\lim_{x\to0}\frac{(5^x-1)\tan x}{\sqrt{x^2+16}-4}\times\frac{\sqrt{x^2+16}+4}{\sqrt{x^2+16}+4}$	1	
		$=\lim_{x\to0}\frac{(5^x-1)\tan x}{x^2+16-16}\times\left(\sqrt{x^2+16}+4\right)$		
		$=\lim_{x\to0}\frac{(5^x-1)\tan x}{x^2}\times\left(\sqrt{x^2+16}+4\right)$	1	
		$=\lim_{x\to0}\frac{5^x-1}{x}\times\frac{\tan x}{x}\times\left(\sqrt{x^2+16}+4\right)$		
		$=\log 5\times1\times\left(\sqrt{0^2+16}+4\right)$	1	
		$=8\log 5$	1	**4**

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>5)</td><td>c)</td><td>Find the root of the equation $x^3-9x+1=0$ which lies between 2 and 3 using Regula-Falsi method.</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>$f(x)=x^3-9x+1$
$\therefore f(2)=-9$
$f(3)=1$
$\therefore$ the root is in $(2,\ 3)$.
$\therefore x_1=\dfrac{2f(3)-3f(2)}{f(3)-f(2)}=2.9$
$\therefore f(2.9)=-0.711$
$\therefore$ the root is in $(2.9,\ 3)$.
$\therefore x_2=2.942$
$\therefore f(2.942)=-0.0139$
$\therefore$ the root is in $(2.942,\ 3)$.
$\therefore x_3=2.943$</td><td>½
½
½
½
½
½
½
½</td><td>4</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(x)=x^3-9x+1$
$\therefore f(2)=-9$
$f(3)=1$
$\therefore$ the root is in $(2,\ 3)$.</td><td>½
½
½</td><td></td></tr>
<tr><td></td><td></td><td><table>
<tr><th>a</th><th>b</th><th>$f(a)$</th><th>$f(b)$</th><th>$x=\dfrac{af(b)-bf(a)}{f(b)-f(a)}$</th><th>$f(x)$</th></tr>
<tr><td>2</td><td>3</td><td>-9</td><td>1</td><td>2.9</td><td>-0.711</td></tr>
<tr><td>2.9</td><td>3</td><td>-0.711</td><td>1</td><td>2.942</td><td>-0.0139</td></tr>
<tr><td>2.942</td><td>3</td><td>-0.0139</td><td>1</td><td>2.943</td><td>---</td></tr>
</table></td><td>1
1
½</td><td>4</td></tr>
<tr><td></td><td></td><td>---</td><td></td><td></td></tr>
<tr><td></td><td>d)</td><td>Find a root of $x^3-9x^2-18=0$ by Newton-Raphson method (carry out 3 iterations).</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>$x^3-9x^2-18=0$
$\therefore f(x)=x^3-9x^2-18$
$\therefore f'(x)=3x^2-18x$
$\therefore f(9)=-18$
$f(10)=82$</td><td>½
½
½</td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$x-\frac{f(x)}{f'(x)}=x-\frac{x^3-9x^2-18}{3x^2-18x}$ ---(*)		
		$=\frac{2x^3-9x^2+18}{3x^2-18x}$ ---(**)	1	
		OR	OR	
		$\frac{xf'(x)-[f(x)]}{f'(x)}=\frac{x(3x^2-18x)-(x^3-9x^2-18)}{3x^2-18x}$ ---(*)		
		$=\frac{2x^3-9x^2+18}{3x^2-18x}$ ---(**)	1	
		Start with $x_0=9$,		
		$\therefore x_1=9.222$	½	
		$x_2=9.212$	½	
		$x_3=9.212$	½	**4**
		Note i) Once the formula (*) is formed, writing the direct values of x_i's is permissible, as we allow it in case of Table Format for either bisection method or regula-falsi method.		
		Note ii) To calculate directly the values of x_i's, students may use the formula (*) instead of formulating the reduced form (**) of (*). This is also considerable. No marks to be deducted. The same is also applicable in the next example.		
		OR		
		$x^3-9x^2-18=0$		
		$\therefore f(x)=x^3-9x^2-18$		
		$\therefore f'(x)=3x^2-18x$	½	
		$\therefore f(9)=-18$	½	
		$f(10)=82$	½	
		$\therefore$ start with $x_0=9$		
		$\therefore x_1=x_0-\frac{f(x_0)}{f'(x_0)}$		
		$=9-\frac{f(9)}{f'(9)}$		
		$=9-\frac{-18}{81}$		
		$=9.222$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $= 9.222 - \frac{f(9.222)}{f'(9.222)}$ $= 9.222 - \frac{0.880}{89.140}$ $= 9.212$	1	
		$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ $= 9.212 - \frac{f(9.212)}{f'(9.212)}$ $= 9.212 - \frac{-0.00947}{88.767}$ $= 9.212$	½	**4**
		--		
	e)	Using Newton-Raphson method, find the approximate value of $\sqrt{10}$ (carry out 3 iterations).		
	Ans.	$Let\ \ x = \sqrt{10} \qquad \therefore x^2 - 10 = 0$ $\therefore f(x) = x^2 - 10$ $\therefore f'(x) = 2x$	½	
		$\therefore f(3) = -1$	½	
		$f(4) = 6$	½	
		$x - \frac{f(x)}{f'(x)} = x - \frac{x^2 - 10}{2x} \qquad ---(*)$ $= \frac{x^2 + 10}{2x} \qquad ---(**)$	1	
		OR	OR	
		$\frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(2x) - (x^2 - 10)}{2x} \qquad ---(*)$ $= \frac{x^2 + 10}{2x} \qquad ---(**)$	1	
		Start with $x_0 = 3$, $\therefore x_1 = 3.167$	½	
		$x_2 = 3.162$	½	
		$x_3 = 3.162$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Note :** If the problem is solved by taking $f(x)=x-\sqrt{10}$, **no marks to be given** since to find various values of $f(x)$ for different values of *x*, it is required to use the value of $\sqrt{10}$ and it is not permissible in this example as here given task is to find its approximate value.		
		OR		
		$Let\ \ x=\sqrt{10}$		
		$\therefore x^2-10=0$		
		$\therefore f(x)=x^2-10$		
		$\therefore f'(x)=2x$	½	
		$\therefore f(3)=-1$	½	
		$f(4)=6$	½	
		$\therefore$ start with $x_0=3$		
		$\therefore x_1=x_0-\frac{f(x_0)}{f'(x_0)}$		
		$=3-\frac{f(3)}{f'(3)}$		
		$=3-\frac{-1}{6}$		
		$=3.167$	1	
		$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$		
		$=3.167-\frac{f(3.167)}{f'(3.167)}$		
		$=3.167-\frac{0.0299}{6.334}$		
		$=3.162$	1	
		$x_3=x_2-\frac{f(x_2)}{f'(x_2)}$		
		$=3.162-\frac{f(3.162)}{f'(3.162)}$		
		$=3.162-\frac{-0.0018}{6.324}$		
		$=3.162$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks

5)

f) Find the root of equation $x^3-4x+1=0$ using bisection method (carry out 3 iterations).

Ans.

$x^3-4x+1=0$

$f(x)=x^3-4x+1$

$\therefore f(0)=1$ ½

$f(1)=-2$ ½

$\therefore$ the root is in $(0,\ 1)$. ½

$\therefore x_1=\frac{0+1}{2}=0.5$ ½

$\therefore f(0.5)=-0.875$ ½

$\therefore$ the root is in $(0,\ 0.5)$.

$\therefore x_2=\frac{0+0.5}{2}=0.25$ ½

$\therefore f(0.25)=0.016$ ½

$\therefore$ the root is in $(0.25,\ 0.5)$.

$\therefore x_3=\frac{0.25+0.5}{2}=0.375$ ½ **4**

OR

$x^3-4x+1=0$

$f(x)=x^3-4x+1$

$\therefore f(0)=1$ ½

$f(1)=-2$ ½

$\therefore$ the root is in $(0,\ 1)$. ½

a	b	$x=\frac{a+b}{2}$	$f(x)$	Marks	Total Marks
0	1	0.5	-0.875	1	
0	0.5	0.25	0.016	1	
0.25	0.5	0.375	---	½	**4**

Note (*) : In **numerical methods problems only**, writing directly the exact values of functions, such as here in this example f(0) or f(1), is allowed.

Note for Numerical Problems: For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus the solution is taken up to 3 decimal points only. Further if answer is truncated

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>5)</td><td></td><td>more than 3 decimal points, the final answer may vary for last decimal point/s. Due to the use of advance calculators, such as modern scientific non-programmable calculators, 1/3 is actually 0.333333333333 but can be taken as 0.333 or in case of 3/7 it is actually 0.428571428 but it is truncated as 0.429. Further it is preferred that in numerical methods the answers are to be in decimal forms, but still many times students keep answers in fractional form. In this case, no marks to be deducted.</td><td></td><td></td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x^3-4x+1=0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(x)=x^3-4x+1$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1)=-2$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(2)=1$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(1,\ 2)$.</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore x_1=\dfrac{1+2}{2}=1.5$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1.5)=-1.625$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(1.5,\ 2)$.</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore x_2=\dfrac{1.5+2}{2}=1.75$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1.75)=-0.641$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(1.75,\ 2)$.</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore x_3=\dfrac{1.75+2}{2}=1.875$</td><td>½</td><td>4</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x^3-4x+1=0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(x)=x^3-4x+1$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1)=-2$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(2)=1$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(1,\ 2)$.</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td><table><tr><th>a</th><th>b</th><th>$x=\dfrac{a+b}{2}$</th><th>$f(x)$</th></tr><tr><td>1</td><td>2</td><td>1.5</td><td>-1.625</td></tr><tr><td>1.5</td><td>2</td><td>1.75</td><td>-0.641</td></tr><tr><td>1.75</td><td>2</td><td>1.875</td><td>---</td></tr></table></td><td>1
1
½</td><td>4</td></tr>
<tr><td></td><td></td><td>---</td><td></td><td></td></tr>
</table>

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**Attempt any FOUR of the following:**		
	a)	Find $\frac{d^2y}{dx^2}$, if $x=a\cos\theta,\quad y=a\sin\theta$		
	Ans.	$x=a\cos\theta,\quad y=a\sin\theta$		
		$\therefore x^2+y^2=a^2\cos^2\theta+a^2\sin^2\theta=a^2\left(\cos^2\theta+\sin^2\theta\right)$		
		$\therefore x^2+y^2=a^2$	1	
		$\therefore 2x+2y\frac{dy}{dx}=0$		
		$\therefore \frac{dy}{dx}=-\frac{x}{y}$	1	
		$\therefore \frac{d^2y}{dx^2}=-\frac{y-x\frac{dy}{dx}}{y^2}$	1	
		$\therefore \frac{d^2y}{dx^2}=-\frac{y-x\left(-\frac{x}{y}\right)}{y^2}$	½	
		$\therefore \frac{d^2y}{dx^2}=-\frac{y^2+x^2}{y^3}$	½	**4**
		OR		
		$x=a\cos\theta,\quad y=a\sin\theta$		
		$\therefore x^2+y^2=a^2\cos^2\theta+a^2\sin^2\theta=a^2\left(\cos^2\theta+\sin^2\theta\right)$		
		$\therefore x^2+y^2=a^2$	1	
		$\therefore 2x+2y\frac{dy}{dx}=0$	1	
		$\therefore x+y\frac{dy}{dx}=0$		
		$\therefore 1+y\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{dy}{dx}=0$	½	
		$\therefore y\frac{d^2y}{dx^2}=-1-\left(-\frac{x}{y}\right)^2$	½	
		$\therefore y\frac{d^2y}{dx^2}=-1-\frac{x^2}{y^2}$		
		$\therefore y\frac{d^2y}{dx^2}=\frac{-y^2-x^2}{y^2}$	½	
		$\therefore \frac{d^2y}{dx^2}=\frac{-y^2-x^2}{y^3}$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**OR**		
		$x = a\cos\theta$	½	
		$\therefore \frac{dx}{d\theta} = -a\sin\theta$		
		$y = a\sin\theta$	½	
		$\therefore \frac{dy}{d\theta} = a\cos\theta$		
		$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$	1	
		$\therefore \frac{d}{d\theta}\left(\frac{dy}{dx}\right) = \cos ec^2\theta$	½	
		$\therefore \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\cdot\frac{d\theta}{dx} \quad or \quad \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}}$		
		$= \cos ec^2\theta \times \frac{1}{-a\sin\theta}$	1	
		$= -\frac{1}{a}\cos ec^3\theta$	½	**4**
		---		
	b)	Solve the following equations by Gauss elimination method: $2x + y + z = 10,\ 3x + 2y + 3z = 18, \quad x + 4y + 9z = 16$		
	Ans.	$\begin{array}{l} 6x+3y+3z=30 \\ 3x+2y+3z=18 \\ \underline{-\quad -\quad -\quad -} \\ 3x+y=12 \end{array} \quad \text{and} \quad \begin{array}{l} 18x+9y+9z=90 \\ x+4y+9z=16 \\ \underline{-\quad -\quad -\quad -} \\ 17x+5y=74 \end{array}$	½ + ½	
		$\begin{array}{l} 15x+5y=60 \\ 17x+5y=74 \\ \underline{-\quad -\quad -} \\ -2x=-14 \end{array}$		
		$\therefore x = 7$	1	
		$\therefore y = 12 - 3x = 12 - 21 = -9$	1	
		$\therefore z = 10 - 2x - y = 10 - 14 + 9 = 5$	1	
		$\therefore \quad \boxed{x = 7, \quad y = -9, \quad z = 5}$		**4**
		OR		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$\begin{array}{r} 4x+2y+2z=20 \\ 3x+2y+3z=18 \\ -\quad-\quad-\qquad- \\ \hline x-z=2 \end{array}$ and $\begin{array}{r} 6x+4y+6z=36 \\ x+4y+9z=16 \\ -\quad-\quad-\qquad- \\ \hline 5x-3z=20 \end{array}$	½ + ½	
		$\begin{array}{r} 3x-3z=6 \\ 5x-3z=20 \\ -\quad+\qquad- \\ \hline -2x=-14 \end{array}$		
		$\therefore x=7$	1	
		$\therefore z=x-2=7-2=5$	1	
		$\therefore y=10-2x-z=10-14-5=-9$	1	
		$\therefore \boxed{x=7,\quad y=-9,\quad z=5}$		**4**
		OR		
		$\begin{array}{r} 6x+3y+3z=30 \\ 6x+4y+6z=36 \\ -\quad-\quad-\qquad- \\ \hline -y-3z=-6 \end{array}$ and $\begin{array}{r} 3x+2y+3z=18 \\ 3x+12y+27z=48 \\ -\quad-\quad-\qquad- \\ \hline -10y-24z=-30 \end{array}$	½ + ½	
		$\begin{array}{r} -10y-30z=-60 \\ -10y-24z=-30 \\ +\qquad+\qquad+ \\ \hline -6z=-30 \end{array}$		
		$\therefore z=5$	1	
		$\therefore y=6-3z=6-15=-9$	1	
		$\therefore x=16-4y-9z=16+36-45=7$	1	
		$\therefore \boxed{x=7,\quad y=-9,\quad z=5}$		**4**
		Note: In the method I, first x is eliminated and then z is eliminated to find the value of y first. Whereas in the method II, first y is eliminated and then z is eliminated to find the value of x first. Similarly in the method III, first z is eliminated and then y is eliminated to find the value of x first. These are just illustrations to get desire solution. But student may follow another order of solution just on this line of solution i. e., to say in the method I, student may first eliminate x and then y to find the value of z first, appropriate marks to be given as per above scheme of marking.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)				
	c)	Solve the following equations by Gauss-Seidal method by taking two iterations. $10x+y+z=12,\quad 2x+10y+z=13,\quad 2x+2y+10z=14$		
	Ans.	$10x+y+z=12$ $2x+10y+z=13$ $2x+2y+10z=14$		
		$\therefore x=\frac{1}{10}(12-y-z)$ $y=\frac{1}{10}(13-2x-z)$ $z=\frac{1}{10}(14-2x-2y)$	1	
		Starting with $x_0=0=y_0=z_0$		
		$x_1=1.2$	½	
		$y_1=1.06$	½	
		$z_1=0.948$	½	
		$x_2=0.999$	½	
		$y_2=1.005$	½	
		$z_2=0.999$	½	**4**
		--		
	d)	Solve the following equations by Jacobi's method by performing two iterations only. $15x+2y+z=18,\quad 2x+20y-3z=19,\quad 3x-6y+25z=22$		
	Ans.	$15x+2y+z=18$ $2x+20y-3z=19$ $3x-6y+25z=22$		
		$\therefore x=\frac{1}{15}(18-2y-z)$ $y=\frac{1}{20}(19-2x+3z)$ $z=\frac{1}{25}(22-3x+6y)$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		Starting with $x_0 = 0 = y_0 = z_0$		
		$x_1 = 1.2$	½	
		$y_1 = 0.95$	½	
		$z_1 = 0.88$	½	
		$x_2 = 1.015$	½	
		$y_2 = 0.962$	½	
		$z_2 = 0.964$	½	**4**
		---		
	e)	Solve by Jacobi's method, carry out two iterations only. $10x + y + 2z = 13, \quad 3x + 10y + z = 14, \quad 2x + 3y + 10z = 15$		
	Ans.	$10x + y + 2z = 13$ $3x + 10y + z = 14$ $2x + 3y + 10z = 15$		
		$\therefore x = \dfrac{13 - y - 2z}{10}$ $y = \dfrac{14 - 3x - z}{10}$ $z = \dfrac{15 - 2x - 3y}{10}$	1	
		Starting with $x_0 = 0 = y_0 = z_0$		
		$\therefore x_1 = 1.3$	½	
		$y_1 = 1.4$	½	
		$z_1 = 1.5$	½	
		$\therefore x_2 = 0.86$	½	
		$y_2 = 0.86$	½	
		$z_2 = 0.82$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)				
	f)	If $y=e^{m\sin^{-1}x}$, prove $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m^2y=0$		
	Ans.	Given $y=e^{m\sin^{-1}x}$		
		$\therefore \log y=m\sin^{-1}x$		
		$\therefore \frac{1}{y}\cdot\frac{dy}{dx}=m\cdot\frac{1}{\sqrt{1-x^2}}$	1	
		$\therefore \sqrt{1-x^2}\cdot\frac{dy}{dx}=my$		
		$\therefore \left(1-x^2\right)\cdot\left(\frac{dy}{dx}\right)^2=m^2y^2$		
		$\therefore \left(1-x^2\right)\cdot 2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\cdot(-2x)=m^2\cdot 2y\cdot\frac{dy}{dx}$	1	
		$\therefore 2\frac{dy}{dx}\left[\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}\right]=2\frac{dy}{dx}\left(m^2y\right)$		
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m^2y$	1	
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$	1	**4**
		--		
		-- -- **Important Note** In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. **In such case, FIRST SEE whether the method falls within the scope of the curriculum**, and THEN ONLY give appropriate marks in accordance with the scheme of marking. -- --		